Exercise 0

Show that the set of all dilation - translation $v \mapsto \lambda v + b$ with $\lambda \neq 0$ of an affine plane form a group under composition.

Let A be set of all dilation transformations

$$v \mapsto \lambda v + b \quad \lambda \neq 0$$

claim: A is a group with operation composition.

Let $D_1, D_2 \in A$

such that $D_1(v) = \lambda v + b, \lambda \neq 0$

$D_2(v) = \lambda' v + b', \lambda' \neq 0$

so ; $(D_2 \circ D_1)v = D_2(D_1(v)) = D_2(\lambda v + b)$

$= \lambda'(\lambda v + b) + b'$

$= \lambda \lambda' v + \lambda' b + b'$

$(D_2 \circ D_1)(v) = (\lambda \lambda') v + (\lambda' b + b')$

$\lambda \neq 0$ and $\lambda' \neq 0$

so $\lambda \lambda' \neq 0$

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Exercise 0

Hence $D_2 \circ D_1 \in A$

So A is close under composition

Now set $D_e \in A$ such that

$$D_e(v) = v$$

So for any $D_i \in A$

$$(D_e \circ D_i)(v) = D_i(D_e(v)) = D_i(v)$$

Hence

$$D_e \circ D_i = D_i \circ D_e = D_i \quad \forall D_i \in A$$

So D_e is the identity element.

set $D \in A$ be any

$$D(v) = \lambda v + b \;,\; \lambda \neq 0$$

Let us define.

$$D^{-1}(v) = \frac{1}{\lambda} v - \frac{b}{\lambda}$$

so clearly $D^{-1} \in A$

$$(D^{-1} \circ D)(v) = D^{-1}(D(v)) = D^{-1}(\lambda v + b)$$

$$= \frac{1}{\lambda}(\lambda v + b) - \frac{b}{\lambda}$$

$$= v = D_e(v)$$

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Exercise o

$$(D \circ D^{-1})(v) = D(D^{-1}(v)) = D\left(\tfrac{1}{\lambda}v - \tfrac{b}{\lambda}\right)$$

$$= v = D_e(v).$$

Hence

$$D \circ D^{-1} = D^{-1} \circ D = D_e.$$

so D^{-1} is the inverse of D

so each element in A has inverse.

Now

Let $D_1(v) = \lambda_1 v + b_1$

$D_2(v) = \lambda_2 v + b_2$

$D_3(v) = \lambda_3 v + b_3$

So

$$D_1 \circ (D_2 \circ D_3) v = D_1(D_2 \circ D_3(v))$$

$$= D_1(D_2(D_3(v)))$$

$$= D_1(\lambda_2 \lambda_3 v + \lambda_2 b_3 + b_2)$$

$$= \lambda_1 \lambda_2 \lambda_3 v + \lambda_1 \lambda_2 b_3 + \lambda_1 b_2 + b_1$$

and $(D_1 \circ D_2) \circ D_3(v) = (D_1 \circ D_2)(D_3(v))$

$$= D_1(D_2(\lambda_3 v + b_3))$$

$$= \lambda_1 \lambda_2 \lambda_3 v + \lambda_1 \lambda_2 b_3 + \lambda_1 b_2 + b_1$$

$$= D_1 \circ (D_2 \circ D_3) v$$

Exercise 0

Hence

$D_1 \circ (D_2 \circ D_3) = (D_1 \circ D_2) \circ D_3$

so, associativity Holds in A

Hence A is a group.

Exercice 1

Let $D_1(v) = 2\left(v - \begin{bmatrix}1\\1\end{bmatrix}\right) + \begin{bmatrix}1\\1\end{bmatrix}$ be dilation in $\mathbb{R}^2$

Let $D_2(v) = \lambda(v-p)+p$

So $D_1(v) = 2\left(v - \begin{bmatrix}1\\1\end{bmatrix}\right) + \begin{bmatrix}1\\1\end{bmatrix}$

$$= 2v - \begin{bmatrix}2\\2\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}$$

$$= 2v - \begin{bmatrix}1\\1\end{bmatrix}$$

Now $D_2 \circ D_1(v) = D_2(D_1(v)) = D_2\left(2v - \begin{bmatrix}1\\1\end{bmatrix}\right)$

$$= \lambda\left(2v - \begin{bmatrix}1\\1\end{bmatrix} - p\right) + p$$

Let $p = \begin{pmatrix}x\\y\end{pmatrix}$

So $D_2 \circ D_1(v) = \lambda\left(2v - \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}x\\y\end{bmatrix}\right) + \begin{bmatrix}x\\y\end{bmatrix}$

$$= 2\lambda v - \lambda\begin{bmatrix}1+x\\1+y\end{bmatrix} + \begin{bmatrix}x\\y\end{bmatrix}$$

$$= 2\lambda v + \begin{bmatrix}x-\lambda-\lambda x\\y-\lambda-\lambda x\end{bmatrix}$$

Exercise 1

but $D_2 \circ D_1(V) = V + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

So,

$$2\lambda V + \begin{bmatrix} x - \lambda - \lambda x \\ y - \lambda - \lambda x \end{bmatrix} = V + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

we get

$$2\lambda = 1 \longrightarrow \lambda = \frac{1}{2}.$$

and $x - \lambda - \lambda x = 1$

$$x - \frac{1}{2} - \frac{1}{2}x = 1$$

$$x = 3$$

$$y - \lambda - \lambda y = 0$$

$$y - \frac{1}{2} - \frac{1}{2}y = 0$$

$$y = \underline{1}$$

Hence $P = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$D_2(V) = \frac{1}{2}\left(V - \begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Exercise 2

F^2

$\mathbb{F} = [(a,b) : a,b \in F_3]$ with $(a,b)+(a'b') = (a+a', b+b')$ and $(a,b)(a'b') = (aa' - bb'd, ab' + a'b)$ d elements

(i) How many point are in $\mathbb{F}^2$?

F_3^2 is a cartesian product $q = 3 \cdot 3$ points

Line is given by $q + \lambda v$ where $v \in F_3$. There are many points on each lines than are element in F_3 i.e 3 points

Pick a point different from origin. There is one line through the origin and it's chosen point

So we have 8 lines $(9-1) = 8$

There are 4 lines through the origin.

Therefore Total lines. (9×8)

$= 72$ lines.

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Exercise 2

Each line is unique $3! = 6$ tim

There are 3 points on each line

Hence $\frac{72}{6} = 12$ lines.

(1 How many points are in F_3^2?

$\frac{72}{2}$ points.

$\frac{72}{2}$ points $= 36$ points

(2 How many lines in F_3^2?

12 lines

(3 How many lines are in F_3^2 that pass through the origin $(0,0)$?

4 lines

(4 How many points lie on each line

3 points on each line